Muscle synergies without a brain or spinal cord

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University of Michigan
in collaboration with
The Rehabilitation Institute of Chicago

Tuesday, July 21, 2009
Multi-muscle mechanics and neural coordination

Muscle activation for movement

Pairing EMG and force fluctuations

Mapping force fluctuations
Multi-muscle mechanics and neural coordination

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Muscle activation for movement
The problem with synergies

No synergies for finger muscles

Where do muscle synergies come from?

Muscle synergies may not be necessary

Towards clinical application
The problem with synergies
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The problem with synergies
The problem with synergies

extension

abduction

flexion

adduction
The problem with synergies
The problem with synergies

- FDI
- LUM
- FDS
- FDP
- EDC
- EIP
- FPI

 Movements:
- Extension
- Abduction
- Flexion
The problem with synergies

Synergy 1

Synergy 2

Synergy 3
The problem with synergies
The problem with synergies
The problem with synergies

Synergy 1

Synergy 2

Synergy 3

Best mechanical strategy - extensors only
The problem with synergies

Best mechanical strategy - extensors only
Best neural strategy - synergy 1 and 2
The problem with synergies

Best mechanical strategy - extensors only

Best neural strategy - synergy 1 and 2

Neuro-mechanical conflict!
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Kutch JJ, Kuo AD, Bloch AM, Rymer WZ. Endpoint force fluctuations reveal flexible rather than synergistic patterns of muscle cooperation. J Neurophysiol 100: 000–000, 2008. First published MONTH; doi:10.1152/jn.90274.2008. We developed a new approach to investigate how the nervous system activates multiple redundant muscles by studying the endpoint force fluctuations during isometric force generation at a multi-degree-of-freedom joint. We hypothesized that, due to signal-dependent muscle force noise, endpoint force fluctuations would depend on the target direction of index finger force and that this dependence could be used to distinguish flexible from synergistic activation of the musculature. We made high-gain measurements of isometric forces generated to different target magnitudes and directions, in the plane of index finger metacarpophalangeal joint abduction–adduction/flexion–extension. Force fluctuations from each target were used to calculate a covariance ellipse, the shape of which varied as a function of target direction. Directions with narrow ellipses were approximately aligned with the estimated mechanical actions of key muscles. For example, targets directed along the mechanical action of the first dorsal interosseous (FDI) yielded narrow ellipses, with 88% of the variance directed along those target directions. If the FDI is likely a prime mover in this target direction and that, at most, 12% of the force variance could be explained by synergistic coupling with other muscles. In contrast, other target directions exhibited broader covariance ellipses with as little as 30% of force variance directed along those target directions. This is the result of cooperation among multiple muscles, based on independent electromyographic recordings. However, the pattern of cooperation across target directions indicates that muscles are recruited flexibly in accordance with their mechanical action, rather than in fixed groupings.

Introduction

The CNS can typically utilize many different muscle combinations when controlling multiple degrees of freedom (DOF) of the body. To simplify task control (Bernstein 1967), it has been proposed that the CNS enforces muscle synergies: fixed patterns of activation among multiple muscles acting about the relevant DOF (d’Avella et al. 2003; Drew et al. 2008; Giuzoter et al. 2007; Ivanenko et al. 2006; Overduin et al. 2008; Saïtel et al. 2001; Ting and Macpherson 2005; Tresch et al. 2006). Alternatively, the CNS can use a task-specific muscle coordination pattern without requiring fixed patterns, perhaps reflecting the optimization of movement according to some suitable performance criteria (Buchanan et al. 1986; Keenan et al. 1994; Wolpert 1998; Krebs et al. 2004; Tudorone and Jordan 2002; Valero-Cuevas 2000; Valero-Cuevas et al. 1998). It is also unclear whether some force in some directions is generated by a “prime mover” muscle (Thomas et al. 1986) or whether all force generation involves the cooperation of multiple muscles (Buchanan et al. 1986; Keenan et al. 2006). These questions remain unresolved, in spite of multiple attempts to characterize muscle activation patterns across multiple DOFs.

The synergistic activation hypotheses and the task-specific flexible activation hypotheses are not incompatible; strategies could apply to voluntary skilled tasks different from those applying to stereotypical reflexive tasks. To separate these hypotheses in voluntary tasks, we introduce a new method for assessing muscle force contributions to net force generation at a multiple DOF joint: the metacarpophalangeal (MCP) joint of the index finger. This method uses high-gain force measurements recorded at the finger tip to estimate the various muscle contributions to net joint force. In contrast, most prior studies investigating muscle coordination across multiple DOFs have focused on the use of electromyographic (EMG) recordings. Although such EMG recordings provide valuable information about muscle activity, they offer significant disadvantages for studying muscle coordination in multiple muscle systems. For example, it is not always possible to record EMGs from all muscles that may contribute to a task. Also, the identification of muscle-level synergies from EMGs in natural behaviors may also be complicated by the existence of biomechanical or task-planning constraints unrelated to muscle synergies. If the CNS chooses to generate force in stereotypical ways, it may cause muscle activation patterns to appear to obey simplifying activation constraints, even if other activation patterns are possible.

An alternative approach to studying muscle coordination involves mapping isometric endpoint force variability for an array of targets distributed uniformly across the endpoint force space. Stochastic effects may enter such tasks in several ways, but one of the most significant is signal-dependent noise (SDN) (Enoka et al. 1999; Galganski et al. 1993; Jorda et al. 2000; Schmidt et al. 1979; Slifkin and Newell 1999), where isometric force variability increases with average isometric force. Such SDN may arise from the sequential recruitment of motor units with larger twitch forces as the muscle force requirement increases (Jones et al. 2002). If muscle force variability increases with average muscle force, then differing neuromotor control strategies can generate different patterns of endpoint force variability. For example, one muscle acting alone will
No synergies for finger muscles
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- Endpoint
No synergies for finger muscles

- Endpoint
- Task space (what the endpoint can do)
No synergies for finger muscles

- Endpoint
- Task space (what the endpoint can do)
- Muscle action (direction of capability)

Westling et al. 1990
No synergies for finger muscles

- Endpoint
- Task space (what the endpoint can do)
- Muscle action (direction of capability)
- Muscle endpoint vector (force applied)
No synergies for finger muscles

- Endpoint
- Task space (what the endpoint can do)
- Muscle action (direction of capability)
- Muscle endpoint vector (force applied)
- Signal-dependent noise (more fluctuation for more force)

Thomas et al. 1991
No synergies for finger muscles

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Thomas et al. 1991

Diagram showing hand with arrows indicating directions:
- up
- right
- left
- down

Graph showing noise (% max force) against force (% max force): after Moritz et al. 2005

Noise (% max force):
- 0
- 20
- 40
- 60
- 80
- 100
- 120

Force (% max force):
- 0
- 20
- 40
- 60
- 80
- 100

Frequency: 8 Hz
No synergies for finger muscles

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- Net endpoint vector (overall force direction)
No synergies for finger muscles

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- Directed ellipse (primary muscle)
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- Force Covariance Map (FCM)
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Details:

1. Load cell (JR3, Woodland, CA) has 1 mN smallest measurable load
2. Load amplified using lightweight aluminum tube
3. Low electrical noise environment
4. Table vibration minimized
5. Surface EMG measured using silver electrodes with self-contained preamplifiers (Delsys, Boston, MA)
6. Subject’s given a target and a force cursor
7. Targets span plane with radial grid
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Task-directed variance fraction

\[ \eta = 95\% \]

\[ \eta = 71\% \]

\[ \eta = 40\% \]

\[ \eta = 10\% \]
No synergies for finger muscles

- Task-confined variance fraction ($\eta$) for different task levels:
  - Task level 1
  - Task level 2
  - Task level 3
- Colors represent different subjects.

- EMG activity and muscle pulling directions comparison:
  - FDI active
  - EIP active
  - EDC active
  - FDP

- Active angles:
  - 0˚, 90˚, 180˚, 270˚, 315˚, 225˚, 135˚, 45˚, 270˚, 315˚, 225˚, 135˚, 45˚, 0˚
No synergies for finger muscles

\[ \eta = 1.0 \]

**Task-confined variance fraction (\( \eta \))**

- Task level 1
- Task level 2
- Task level 3

Colors: subjects

Muscle pulling directions:
- FDI active
- EIP active
- EDC active
- FDP active

TDC-TDC-

FDI 225˚ 0˚ 45˚ 135˚ 90˚ 180˚ 270˚ 315˚ 225˚

FDL-SM 0˚ 45˚ 135˚ 90˚ 180˚ 270˚ 315˚

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The problem with synergies

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Where do muscle synergies come from?

Muscle synergies may not be necessary

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Where do muscle synergies come from?
Computer-controlled cadaver hand

A. Experimental preparation

- Motors and encoders (not pictured)
- Load cells
- Nylon cords
- Motion capture markers

B. Recording mode

- Movement between postures 1 and 2

C. Playback mode

Tapping

Scratching

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Where do muscle synergies come from?

Torres-Oviedo et al. 2006

d’Avella et al. 2006
Where do muscle synergies come from?

(a) rectified, filtered, aligned
(b) averaged and tonic
(c) phasic
(d) normalized

Muscle abbreviations:
- BicShort
- BicLong
- Brac
- PronTer
- BrRad
- TrLat
- TrLong
- TrMed
- DeltA
- DeltM
- DelTR
- PectClav
- TrapSup
- TrapMed
- LatDors
- TeresMaj
- InfraSp
- end-point speed

Graphs showing various muscle responses with time and voltage scales.
Where do muscle synergies come from?

Ting and Macpherson, 2005
Where do muscle synergies come from?
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Biomechanical constraints play under-appreciated role in observing muscle synergies
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Task commands

abd/add

flex/ext

Minimum Effort

1 2 ... 7 Muscles
Muscle synergies may not be necessary

Reasoning:

Task commands:
- abd/add
- flex/ext

Minimum Effort

Muscles:
- 1
- 2
- ...
- 7
Muscle synergies may not be necessary

Reasoning:
1. Todorov (2002): minimize squared muscle inputs gives rise to cosine tuning
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3. Put 2 & 2 together: tuned motoneurons could minimize effort.
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Minimum effort solution with simpler neural network than synergies.
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Reasoning:
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**Minimum effort solution with simpler neural network than synergies.**
Muscle synergies may not be necessary

Diagram:
- abd/add
- flex/ext
- Task commands
- Muscles

Numbers:
- 0.0175
- 0.0040
- -0.0128
Muscle synergies may not be necessary

- Task commands
- Muscles
- FDI, EDC, EIP
- FDP, FDS, LUM, FPI
- Minimum effort
- Network generated

Tuesday, July 21, 2009
The problem with simplification

No synergies for finger muscles

Flexible activity noise vs synergy noise

Muscle synergies may not be necessary

Towards clinical application
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Towards clinical application
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The Challenge: Dispense with needles and intramuscular electrodes

Patient: Doe, J.               6-01-2011
Towards clinical application

Dyno-myography:
Towards clinical application

Dyno-myography:

A. Many different muscle activity combinations produce the same desired average net force vector.
Towards clinical application

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B. But different muscle activity combinations produce different force covariance ellipses
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Extensors

Flexors

FDP

FPI
Towards clinical application

C
Hypertonic muscle: flexors have a minimum activity for all tasks
Inability to deactivate appears in tuning curve

FDI Extensors
Flexors FPI

D
Weak muscle: flexors can not generate substantial force
Muscle can not produce force

Other muscles compensate

FDI Extensors
Flexors FPI

Hypertonic muscle: flexors have a minimum activity for all tasks

Normal:
minimal muscle activity for the task

Abnormal synergy:
activity in FDI and flexors are linked

Hypertonic muscle: flexors have a minimum activity for all tasks

Weak muscle: flexors can not generate substantial force

Hypertonic muscle: flexors have a minimum activity for all tasks

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1. Muscle synergies can interfere with good mechanical choices
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2. Noise: “index finger redundancy not resolved muscle synergy”
Conclusions

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3. Muscle synergies could be generated by biomechanics
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4. Simple neural network can give minimum effort for all tasks
Conclusions

1. Muscle synergies can interfere with good mechanical choices

2. Noise: “index finger redundancy not resolved muscle synergy” (maybe minimum effort)

3. Muscle synergies could be generated by biomechanics

4. Simple neural network can give minimum effort for all tasks

5. Future: non-invasive study of multiple muscle systems
Acknowledgments

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Heiko Hoffmann
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Flexible activity noise vs. synergy noise
Flexible activity noise vs. synergy noise

\[
\begin{align*}
\tau & = Af \\
\bar{\tau} & = A\bar{f} \\
\sigma_i & = k_i \bar{f}_i^{\alpha_i} \\
\text{cov}[\tau] & = A(\sigma\sigma^T \ast P)A^T
\end{align*}
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Flexible activity noise vs. synergy noise

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moment arms transform muscle forces to joint torques
Flexible activity noise vs. synergy noise

\[ \tau = Af \quad \text{moment arms transform muscle forces to joint torques} \]
\[ \tilde{\tau} = A\tilde{f} \quad \text{expectation (averaging) is linear} \]
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\[ \text{cov}[\tau] = A (\sigma \sigma^T \ast P) A^T \quad \text{covariance of a linear transformation is a quadratic form} \]
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A. Fowler et al. (2001) moment arms
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\text{cov} \left[ \tau \right] &= A \left( \sigma \sigma^T \cdot P \right) A^T \quad \text{signal-dependent noise follows a power law} \\
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expectation (averaging) is linear

\[ \sigma_i = k_i \bar{f}_i^{\alpha_i} \]  
signal-dependent noise follows a power law

\[ \text{cov}[\tau] = A(\sigma\sigma^T \cdot P)A^T \]  
covariance of a linear transformation is a quadratic form

A. Fowler et al. (2001) moment arms

minimize sum of squared muscle forces (quadratic programming)
Flexible activity noise vs. synergy noise

\[ \tau = Af \]  
\[ \bar{\tau} = A\bar{f} \]
moment arms transform muscle forces to joint torques

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Spans the plane  

Do not span the plane  

Tuesday, July 21, 2009
Flexible activity noise vs. synergy noise

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Flexible activity noise vs. synergy noise

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Tuesday, July 21, 2009
Flexible activity noise vs. synergy noise
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A. Fowler et al. (2001) moment arms

B. Synergy activation (syn.)

C. Minimum effort activation (min.)

Tuesday, July 21, 2009
Flexible activity noise vs. synergy noise

A. Fowler et al. (2001) moment arms

B. Synergy activation (syn.)

C. Minimum effort activation (min.)

Task commands

Muscles

Syn 1  Syn 2  Syn 3

Synergies

1  2  …  7

Muscles

abd/add  flex/ext

ext  flex  add  ext

abd  add

0.9

Wednesday, July 22, 2009
Flexible activity noise vs. synergy noise

A. Fowler et al. (2001) moment arms

B. Synergy activation (syn.)

1 2 3

Muscles

Syn 1  Syn 2  Syn 3

Synergies

abd/add  flex/ext

Task commands

C. Minimum effort activation (min.)

abd/add  flex/ext

Minimum Effort

Task commands

Muscles

Tuesday, July 21, 2009
Flexible activity noise vs. synergy noise

A. Fowler et al. (2001) moment arms

B. Synergy activation (syn.)

C. Minimum effort activation (min.)

H. Random search histograms

Task commands

Muscles

Syn 1  Syn 2  Syn 3

Synergies

abd/add  flex/ext

1  2  …  7

Muscles

ext

ext

abd/add  flex/ext

abd  add

EDC  EIP

FDI  FDP

LUM  FDS

Minimum Effort

Minimum Effort

abd/add  flex/ext

1  2  …  7

Muscles

Syn.

Min.

Data

Count

0.4  0.6  0.8

Average η in peak data directions

Tuesday, July 21, 2009
C. Dyno-myography estimates muscle tuning curves by only selecting muscle activity combinations consistent with the force covariance ellipse.